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## Letters

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### Comments on "Integration Method of Measuring $Q$ of the Microwave Resonators"

P. L. OVERFELT AND D. J. WHITE

The paper of this title<sup>1</sup> suggested integrating the power

$$P(\omega) = P_0 \left[ 1 + Q_L^2 (\omega/\omega_0 - \omega_0/\omega)^2 \right]^{-1} \quad (1)$$

transmitted through a resonator [2] across some frequency band in the vicinity of resonance in order to determine the loaded  $Q(Q_L)$ ; it pointed out some advantages over the common approach using the one-half power frequencies. The integrated power was found [1] to be

$$I = \int_{\omega_1}^{\omega_2} P(\omega) d\omega = P_0 \omega_0 Q_L^{-1} \tan^{-1} (Q_L \omega_s \omega_0^{-1}) \quad (2)$$

where  $\omega_s = \omega_2 - \omega_1$ .

Since we have experienced the usual frustrations in  $Q$ -measurement, a new approach is attractive, the first step of which is to check the derivations. This turned out to be more difficult than

anticipated, and initially we were unable to come up with a reasonable derivation of (2). However, (1) can be integrated exactly by writing the integral in (2) in the form [3]

$$I/P_0 \omega_0 = (g/h) \int_{x_1}^{x_2} (cx^2 + g)^{-1} dx - (f/h) \int_{x_1}^{x_2} (cx^2 + f)^{-1} dx \quad (3)$$

where  $x = \omega/\omega_0$  and  $c, g, f$ , and  $h$  are appropriate constants. When this is done, we find

$$I = \frac{P_0 \omega_0}{2j Q_L \sqrt{(4Q_L^2 - 1)}} \left[ \left( 1 + j\sqrt{4Q_L^2 - 1} \right) \tan^{-1} \frac{\omega \left( 1 - j\sqrt{4Q_L^2 - 1} \right)}{2Q_L \omega_0} - \left( 1 - j\sqrt{4Q_L^2 - 1} \right) \tan^{-1} \frac{\omega \left( 1 + j\sqrt{4Q_L^2 - 1} \right)}{2Q_L \omega_0} \right]_{\omega_1}^{\omega_2} \quad (4)$$

where we have used the relation

$$\left( 1 - 2Q_L^2 \pm \sqrt{1 - 4Q_L^2} \right)^{1/2} = \left( 1 \pm j\sqrt{4Q_L^2 - 1} \right) / \sqrt{2}. \quad (5)$$

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<sup>1</sup>I. Kneppo, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, no. 2, Feb. 1978.

TABLE I  
 $f_0 = 10 \text{ GHz}$ ;  $P_0 = 1$

$\omega_s, \text{ MHz}$	$Q_L = 10^3$ ; $\Delta f = 10 \text{ MHz}$			$Q_L = 10^4$ ; $\Delta f = 1 \text{ MHz}$			
	$I$ eq. (2)	$I$ eq. (9)	$I$ eq. (7)	$\omega_s, \text{ MHz}$	$I$ eq. (2)	$I$ eq. (9)	$I$ eq. (7)
1	$0.63 \times 10^7$	$0.62 \times 10^7$	$0.62 \times 10^7$	1	$4.93 \times 10^6$	$3.48 \times 10^6$	$3.48 \times 10^6$
3	$1.83 \times 10^7$	$1.70 \times 10^7$	$1.70 \times 10^7$	3	$7.85 \times 10^6$	$4.42 \times 10^6$	$4.42 \times 10^6$
5	$2.91 \times 10^7$	$2.47 \times 10^7$	$2.47 \times 10^7$	5	$8.63 \times 10^6$	$4.62 \times 10^6$	$4.62 \times 10^6$
10	$4.93 \times 10^7$	$3.48 \times 10^7$	$3.48 \times 10^7$	10	$9.24 \times 10^6$	$4.78 \times 10^6$	$4.78 \times 10^6$
20	$1.85 \times 10^7$	$4.42 \times 10^7$	$4.42 \times 10^7$	20	$9.66 \times 10^6$	$4.88 \times 10^6$	$4.88 \times 10^6$
50	$8.63 \times 10^7$	$4.62 \times 10^7$	$4.62 \times 10^7$	50	$9.74 \times 10^6$	$4.90 \times 10^6$	$4.90 \times 10^6$
100	$9.24 \times 10^7$	$4.78 \times 10^7$	$4.78 \times 10^7$	100	$9.81 \times 10^6$	$4.92 \times 10^6$	$4.92 \times 10^6$
200	$9.56 \times 10^7$	$4.86 \times 10^7$	$4.86 \times 10^7$	200	$9.84 \times 10^6$	$4.93 \times 10^6$	$4.93 \times 10^6$
1000	$9.81 \times 10^7$	$4.92 \times 10^7$	$4.92 \times 10^7$	1000	$9.86 \times 10^6$	$4.93 \times 10^6$	$4.93 \times 10^6$

Despite the complex arguments in (4),  $I$  must be real, and this can be confirmed by combining the arc tangents in  $\omega_1$  and  $\omega_2$  separately and using the identity

$$a \tan^{-1} f_1 = j \ln f_2 \quad (6)$$

where

$$f_1 = j(f_2^{2/a} - 1)/(f_2^{2/a} + 1)$$

or

$$f_2 = [(1 + f_1/j)/(1 - f_1/j)]^{a/2}.$$

This results in the following exact expression for  $I$

$$I = \frac{P_0 \omega_0}{2 Q_L} \left\{ \tan^{-1} \left[ \omega_0 Q_L \frac{\omega_2(\omega_0^2 - \omega_1^2) - \omega_1(\omega_0^2 - \omega_2^2)}{(\omega_0^2 - \omega_1^2)(\omega_0^2 - \omega_2^2) Q_L + \omega_1 \omega_2 \omega_0^2} \right] - \frac{1}{2\sqrt{4Q_L^2 - 1}} \ln \left[ \frac{(\omega_0^2 + \omega_2^2) Q_L + \omega_2 \omega_0 \sqrt{4Q_L^2 - 1}}{(\omega_0^2 + \omega_2^2) Q_L - \omega_2 \omega_0 \sqrt{4Q_L^2 - 1}} \right] \right. \\ \left. \cdot \frac{(\omega_0^2 + \omega_1^2) Q_L - \omega_1 \omega_0 \sqrt{4Q_L^2 - 1}}{(\omega_0^2 + \omega_1^2) Q_L + \omega_1 \omega_0 \sqrt{4Q_L^2 - 1}} \right\}. \quad (7)$$

Since (1) is based on an RLC equivalent circuit and is thus presumably valid only for high  $Q$  circuits over a limited bandwidth, we can make the following assumptions which are in accordance with general microwave practice:  $Q_L \gg 1$ ,  $\omega_0 \gg \omega_s$ , and symmetrical integration limits,  $\omega_2 = \omega_0 + \omega_s/2$  and  $\omega_1 = \omega_0 - \omega_s/2$ . Then (7) becomes approximately

$$I = \frac{P_0 \omega_0}{2 Q_L} \left( \tan^{-1} \frac{2 \omega_s Q_L}{\omega_0} - \frac{1}{2 Q_L} \ln \frac{4 \omega_0 + \omega_s}{4 \omega_0 - \omega_s} \right). \quad (8)$$

If the  $\ln$  term is expanded in a Taylor series, it is obviously negligible with respect to the arc tan term for reasonable values of  $\omega_0$ ,  $\omega_s$ , and  $Q_L$ . Hence

$$I = 0.5 P_0 \omega_0 Q_L^{-1} \tan^{-1} (2 \omega_s Q_L / \omega_0). \quad (9)$$

Equation (9) looks very much like (2) except for the factors of 0.5 and 2. However,  $0.5 \tan^{-1} 2a \neq \tan^{-1} a$  in general and Table I compares values computed from (2), (7), and (9) for different values of  $Q_L$ ,  $f_0$ , and  $f_s$ . Equations (9) and (7) are in essentially exact agreement for the parameters used, while the results for (2) are quite different. For a sufficiently small  $Q_L \omega_s \omega_0^{-1}$ , it agrees

well enough; for a large value of this product, the arc tangent goes to  $\pi/2$  and (2) becomes, in the limit, twice the value of (9).

It is instructive to note that if we set  $Q_L = f_0/\Delta f$  and  $\omega_s = 2\pi f_s = 2\pi k \Delta f$ , where  $\Delta f$  is the 3-dB bandwidth and  $k$  some constant, (9) can be rewritten as

$$I = \pi P_0 \Delta f \tan^{-1} 2k. \quad (10)$$

Hence, for a given  $P_0$ ,  $I$  depends only on the 3-dB bandwidth and the ratio of the integration bandwidth to the 3-dB bandwidth.

Equation (2) can be derived by making the approximations leading from (7) to (9) before the actual integration. Setting  $\omega = \omega_0 + \delta\omega$ ,  $\delta\omega \ll \omega_0$ , we have

$$1 + Q_L^2 (\omega/\omega_0 - \omega_0/\omega)^2 \approx 1 + 4Q_L^2 \delta\omega^2/\omega_0^2. \quad (11)$$

Since  $\delta\omega = \omega - \omega_0$ , (11) becomes

$$1 + Q_L^2 (\omega/\omega_0 - \omega_0/\omega)^2 \approx 4Q_L^2 \omega_0^{-2} \omega^2 - 8Q_L^2 \omega_0^{-1} \omega + 4Q_L^2 + 1 \quad (12)$$

and (2) can be integrated directly from a standard integral table as

$$I = P_0 \omega_0 Q_L^{-1} \tan^{-1} (Q_L \omega_s \omega_0^{-1}) \quad (13)$$

having noted that  $\tan^{-1}(-a) = -\tan^{-1}a$ , with (13) and (2) being identical.

We are puzzled by the excellent experimental results in [1]. Since the cavity  $Q$ 's were high and the error terms in [1, table I] are small,  $\tan^{-1}(\omega_s Q_L \omega_0^{-1}) \approx \tan^{-1}(2\omega_s Q_L \omega_0^{-1}) = \pi/2$  and  $Q_L = Q'_L$  with

$$Q'_L = 0.5 \pi P_0 \omega_0 k^{-1} I_t^{-1} \quad (14)$$

from [1], and

$$Q'_L = 0.25 \pi P_0 \omega_0 k^{-1} I_t^{-1} \quad (15)$$

from our derivation ( $k$  and  $I_t$  are defined in [1]). Thus on the surface it would appear that Kneppo [1] should have obtained twice the actual  $Q_L$ . It is not clear whether the discrepancy lies in the measurement process or whether, in this case, an approximation [1, (12)] to an approximation (1) is a more accurate representation of a microwave resonator than the approximation itself [1, (1)].

*Reply<sup>2</sup> by I. Kneppo<sup>3</sup>*

The power

$$P(\omega) = P_0 [1 + Q_L^2 (\omega/\omega_0 - \omega_0/\omega)^2]^{-1} \quad (1)$$

can be integrated using the substitution

$$x = \omega/\omega_0 - \omega_0/\omega. \quad (2)$$

From (2) it follows immediately

$$\omega = (x \omega_0)/2 \pm \left[ ((x \omega_0)/2)^2 + \omega_0^2 \right]^{1/2}$$

and

$$d\omega = \omega_0/2 [1 \pm x(x^2 + 4)^{-1/2}] dx$$

<sup>2</sup>Manuscript received February 7, 1983.

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so we have the integral of (1) in the form

$$I = \int_{\omega_1}^{\omega_2} P(\omega) d\omega = (P_0 \omega_0) / 2 \left[ \int_{x_1}^{x_2} (1 + Q_L^2 x^2)^{-1} dx \right. \\ \left. \pm \int_{x_1}^{x_2} x (1 + Q_L^2 x^2)^{-1} (x^2 + 4)^{-1/2} dx \right] \quad (3)$$

where  $x_1 = \omega_1 / \omega_0 - \omega_0 / \omega_1$  and  $x_2 = \omega_2 / \omega_0 - \omega_0 / \omega_2$ . Limits of the integration fulfill the relations  $x_1 < 0$  and  $x_2 > 0$ , if assumptions  $\omega_1 < \omega_0$  and  $\omega_2 > \omega_0$  are valid. The function in the second integral in (3) is odd, so the value of this integral can be made zero choosing the symmetrical limits of the integration  $x_1 = -x_2$ .

Hence

$$I = (P_0 \omega_0) / 2 \int_{-x_2}^{x_2} (1 + Q_L^2 x^2)^{-1} dx = P_0 \omega_0 \int_0^{x_2} (1 + Q_L^2 x^2)^{-1} dx \\ = P_0 \omega_0 Q_L^{-1} \tan^{-1}(Q_L x_2). \quad (4)$$

Using notation  $\omega_s = \omega_2 - \omega_1$ ,  $\omega_s \ll \omega_0$ , we have  $x_2 \approx \omega_s / \omega_0$ , and (4) has the final form

$$I = P_0 \omega_0 Q_L^{-1} \tan^{-1}(Q_L \omega_s \omega_0^{-1}).$$

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- [3] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 4th ed. New York: Academic, 1965, sec. 2.161, no. 3, p. 67.

#### Corrections to "A Quasi-Optical Polarization Duplexed Balanced Mixer for Millimeter-Wave Applications"

KARL STEPHAN, NATALINO CAMILLERI, AND TATSUO ITOH

In the above referenced paper<sup>1</sup>, the following corrections should be made.

On page 166, the first full sentence should read, "...using the saddlepoint equations given in [4]..."

On page 166, equation (21) should be

$$P = \iint_{\text{sphere}} \frac{\frac{1}{2}(|E_\theta|^2 + |E_\phi|^2)}{Z_{fs}} ds.$$

On page 168, the first sentence of the second full paragraph should read, "Using the measured antenna patterns for the 10-GHz model, an approximate directivity on the dielectric side was calculated to be 5.5 dB..."

On page 169, Table II, line 10 should read "Measured conversion loss: 6.5 dB  $\pm$  3 dB."

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<sup>1</sup>K. Stephan, N. Camilleri and T. Itoh, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-31, pp. 164-170, Feb. 1983.